Hochschule für Angewandte Wissenschaften Hamburg
Hamburg University of Applied Sciences

Technische Universität Berlin

Federal Ministry for Economic Affairs and Energy

Development of the new MINLP Solver Decogo using SCIP - Status Report

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SCIP Workshop 2018, Aachen

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Introduction

Motivation and Goal

Motivation:

1. Branch&Bound is currently the standard MINLP approach, but search-tree can grow rapidly
2. Experience with parallel Column Generation (Inner Approximation) for solving huge crew scheduling problems with 100.000.000 variables, but duality gap must be small
3. Adaptive Outer Approximation (OA) faster than B&B for some special nonconvex MINLPs, e.g. separable, but number of binary variables can be large

Goal:

Implementation of the nonconvex MINLP solver **Decogo** using:

1. Automatic Decomposition : for block-separable reformulation
2. Adaptive OA: for solving medium-scale MINLPs
3. Parallel Inner/Outer Refinement: for solving large-scale MINLPs

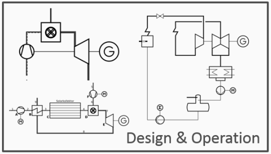
Complex optimization problems with a modular structure

1. Energy system planning

Modules: system components

Subproblem: MINLP under uncertainty

(project 2017-2020 with TU-Berlin)



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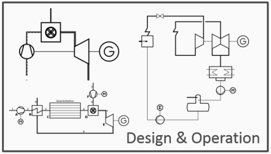
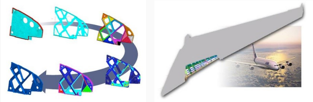


2. Topology optimization

Modules: system components

(e.g. wing rib)

Subproblem: PDE-constrained MIQQP



Altair Hyperworks

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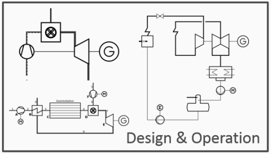

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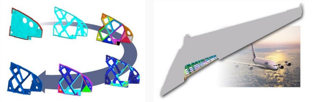

Altair Hyperworks

3. Crew scheduling (LH Systems)

Modules: crew members

Subproblem: constrained shortest path

Decogo (Decomposition-based Global Optimizer)

- **Decogo** is written in Python and uses the modeling and optimization framework **Pyomo**.
- **Ipopt** is used for solving for NLPs, LPs and **SCIP** for MINLPs, MIPs

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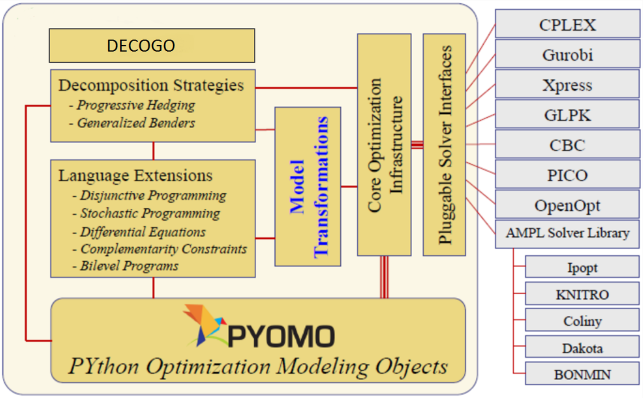


Project and References

Decogo is developed in the project (2017-2020): **MINLP-Optimization of Design and Operation of Complex Energy Systems**

- HAW-Hamburg (V. Gintner, P. Muts, I. Nowak):
  development of Decogo
- TU-Berlin (S. Bruche, E. Papadis, G. Tsatsaronis):
  development of energy system models under uncertainty
- Mitsubishi Hitachi Power Systems Europe GmbH (A. Heberle):
  providing data and consultation

References:

- *Decomposition-based Inner- and Outer-Refinement Algorithms for Global Optimization*, I. Nowak, N. Breitfeld, E. Hendrix, and G. Njacheun-Njanzoua, JOGO, 2018
- *Column Generation based Alternating Direction Methods for solving MINLPs*, I. Nowak, www.optimization-online, 2015

- For more information see: www.researchgate.net/project/Decomposition-Methods-for-Mixed-Integer-Nonlinear-Optimization

Automatic decomposition

Block-separable MINLP reformulation

Goal: reformulation of a sparse optimization problem as
a block-separable MINLP with arbitrary block-sizes

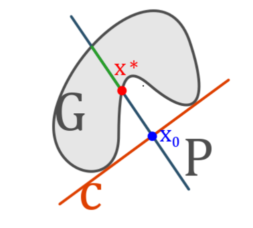
$$\min\{c^T x : x \in P \cap G\} \quad , \qquad G := \prod_{k \in K} G_k$$

with linear global constraints:

$$P := \{x \in \mathbb{R}^n : Ax \le b\}$$

nonlinear local (nonconvex) constraints:

$$G_k := \{y \in \mathbb{R}^{n_{k1}} \times \mathbb{Z}^{n_{k2}} : g_j(y) \le 0, \, j \in J_k\}$$



Basic steps

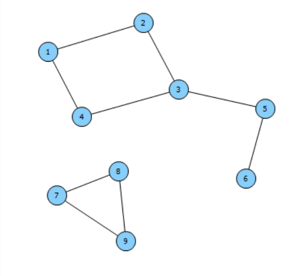
X - set of variables, K - number of blocks

1. Divide the set of variables X into K disjoint subsets with arbitrary size
2. Reformulate the problem by adding new variables with following requirements:
    - Objective function is linear
    - Global constraints are linear
    - Nonlinear constraints are local

Blocks partition: connected components

$$G = (V, E),$$
$$V = \{v_i : v_i \in X\},$$
$$E = \{e_{ij} : e_{ij} = (x_i, x_j), \frac{\partial^2 f_m}{\partial x_i \partial x_j} \neq 0\},$$

where $\frac{\partial^2 f_m}{\partial x_i \partial x_j}$ is Hessian of Lagrangian.



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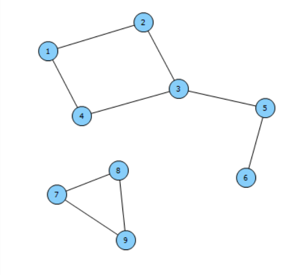

Advantages:

- Easy implementation
- Less number of new variables for the reformulation

Disadvantage:

- Impossible to divide the set of variables with a given block size

Blocks partition: new idea

New weighted graph (or hypergraph):

$$G = (V, E),\ V = \{v_i : v_i \in X\},\ E = \{e_{ij} : e_{ij} = (x_i, x_j), \frac{\partial^2 f_m}{\partial x_i \partial x_j} \neq 0\},$$

$$w(v_i) = W_i\ ,\quad W_i = \{w_i \in \mathbb{N} : \text{number of terms in } \frac{\partial^2 f_m}{\partial^2 x_i}\},$$

$$w(e_{ij}) = W_{ij}\ ,\quad W_{ij} = \{w_{ij} \in \mathbb{N} : \text{number of terms in } \frac{\partial^2 f_m}{\partial x_i \partial x_j}\}.$$

The weights are possibly multiplied by some factors.

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The weights are possibly multiplied by some factors.

Minimum k-cut partition:

$$\min \left[\sum_{i=1}^{k-1} \sum_{j=i+1}^{k} \sum_{\substack{n: v_n \in c_i \\ m: v_m \in c_j}} w(e_{nm}) + \sum_{i=1}^{k} \sum_{v_j \in c_i} w(v_j) \right],$$

where $C = \{c_1, ..., c_k\}$ disjoint subsets of V.

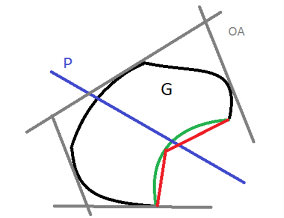
Adaptive Outer Approximation

Convex-Concave Reformulation and MIP Outer-Approximation

Reformulation of MINLP as a Convex-Concave Program (CCP) by replacing nonconvex constraint functions by a DC-formulation

$$g(x) \leq 0 \quad \Leftrightarrow \quad g(x) + \sigma(\|x\|^2 - t) \leq 0, \quad t - \|x\|^2 \leq 0$$

where the convexification parameters σ are updated dynamically



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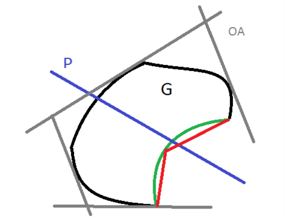
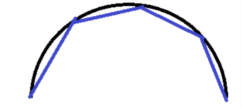
MIP outer approximation:

- replace convex functions by linear approximations $\rightarrow \check{C}$
- replace separable concave functions by piecewise linear functions $\rightarrow \check{Q}_B$
- MIPOA:
  $\min c(x) : x \in \check{P}, \quad (x, t) \in \check{C} \cap \check{Q}_B$



Adaptive MIP based OA-Solver

The OA-Solver solves a CCP
by successively updating MIPOAs using a
limited number of breakpoints



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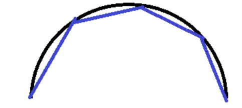

adaptSolveCCP

1. compute a solution of the MIPOA
2. project solution onto feasible set G
3. add linearization cuts to $\check{C}$
4. update breakpoints of $\check{Q}_B$ and convexification parameters σ

The OA-Solver is used for

- solving medium-scale MINLP-subproblems and for
- computing lower bounds of the (large-scale) original MINLP

Inner- and Outer-Refinement

LP inner and outer approximation

CG successively improves the
LP inner-approximation (IA) (restricted master problem)

$$x_0 = \operatorname{argmin}\{c^T x : x \in P \cap \operatorname{conv}(S)\}$$

by adding points to a sample set $S \subset G$ by solving MINLP subproblems.

The IA is equivalent to

$$\min\{c^T x(z) : x(z) \in P,\ z \in \prod_{k \in K} \Delta_{|V_k|}\}, \quad x(z) := \sum_{v \in V_k} \hat{y}_{kv} z_{kv}$$

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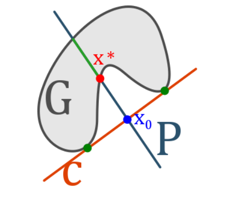
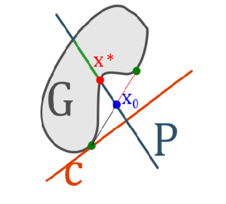
z_{kv} is called L-score.
$z_{kv} \sim 1/\mathrm{dist}(y_{kv}, x_{0k})$, i.e. if $z_{kv} \simeq 1$, then $\hat{y}_{kv} \simeq x_{0k}$



left: similar L-scores
right: different L-scores
duality gap $c^T x^* - c^T x_0$
is smaller



LP inner and outer approximation

LP outer-approximation (OA)

$$\min\{c^T x : x \in \check{P}\}$$

where $\check{P} \supset P \cap G$ is a polyhedron

CG solves the convex relaxation (CR):

$$\min\{c^T x : x \in P \cap \text{conv}(G)\}$$

and generates inner and outer approximations.

Decomposition-based Inner- and Outer-Refinement (Dior)

1. $\check{P} \leftarrow$ autoDecomp
2. $\check{C} \leftarrow$ initCCP($\check{P}$)
3. $(\check{P}, \check{C}) \leftarrow$ reduceBox($\check{P}, \check{C}$) `# reduce bounding box`
4. $(z, S, \check{P}, \check{C}) \leftarrow$ colGenStart($\check{P}, \check{C}$) `# init IA/OA`
5. $(\underline{v}, \overline{v}, x^*) \leftarrow$ outerMipHeu $(x(z), \check{P}, \check{C})$ `# compute sol.`
6. Repeat
    - 6.1 $(\check{P}, \check{C}) \leftarrow$ addOptCuts $(x^*, \check{P}, \check{C})$ `# cut off sol.`
    - 6.2 $(z, S, \check{P}, \check{C}) \leftarrow$ addLagCuts $(S, \check{P}, \check{C})$ `# improve IA/OA`
    - 6.3 $(\underline{v}, \overline{v}, x^*) \leftarrow$ outerMipHeu $(x(z), \check{P}, \check{C})$ `# compute sol.`
    - 6.4 Until $\overline{v} - \underline{v} < \epsilon$ or stopping criterion
7. Return $(\underline{v}, \overline{v}, x^*)$

Preliminary results with column generation

Example 1 from MINLPLib: pooling problem[1]

All experiments were performed on Windows-based computer with 16 GB RAM and Intel Core i7-7820HQ CPU running at 3.8 GHz

Reformulation:

- The problem has 3 blocks with size 9 in Hessian of Lagrangian.
- After reformulation: $K = 4$ blocks with block sizes (21, 21, 21, 5).

Comparison of original problem and reformulated:

	original problem	after reformulation
Number of variables	32	68
Linear constraints	8	19
Nonlinear constraints	11	36

[1] http://www.minlplib.org/ex5_2_5.html

Example 1 from MINLPLib: pooling problem

Best known solutions:

Primal bound	Dual bound
-3500	-3500.000004 (ANTIGONE)
	-6233.265793 (BARON)
	-7496.090988 (SCIP)

Our result:

- Estimate of the lower bound = val(OA) = -13330.3708 in 48s.
- Estimate of the upper bound = val(IA) = -3373.71873
- Gap reported by SCIP = 8857.92
- Convergence of SCIP is slow $\Rightarrow$ termination after 10000 nodes
- val(OA) = -3671.3708 in $\approx$ 9 hours.

Example 2 from MINLPLib: QCP[2]

Reformulation:

- The problem has 16 blocks with average blocksize in Hessian of Lagrangian 7.5625.
- After reformulation: $K = 17$ blocks with average block size 23.

Comparison of original problem and reformulated:

	original problem	after reformulation
Number of variables	126	391
Linear constraints	28	93
Nonlinear constraints	65	265

[2] http://www.minlplib.org/ex8_3_8.html

Example 2 from MINLPLib

Best known solutions:

Primal bound	Dual bound
-3.25611909	-5.70013654 (ANTIGONE)
	-10.00000000 (BARON)
	-10.00000000 (SCIP)

Our result:

- Estimate of the lower bound = val(OA) = -10.0 in 369s.
- Estimate of the upper bound = val(IA) = -9.6
- Gap reported by SCIP = 20676.41
- Convergence of SCIP is slow $\Rightarrow$ termination after 10000 nodes

Final Remarks

Remarks

Dior:

- new exact MINLP approach, not based on B&B
- motivated by CG for huge transport optimization problems
- successive approximation method, which iteratively finds better points by improving inner- and outer-approximations

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- parallel solving (small difficult) sub-problems
- general approach for modular/sparse problems

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Next steps:

- finish implementation of Decogo (new project)
- solve energy system planning and topology optimization problems

Questions?